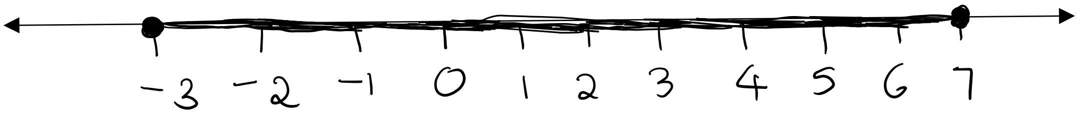
1. 2
2. 5
3. 8
4. [-7,3]
5. $x + y > 2\sqrt{xy}$
6. <
7. X= 3 , y= -4
8. $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$
9. 108
10. 3 or$\frac{3}{2}$
11. -1
12. -17
13. 126
14. $x + y = 1$
15. $local\ minima$
16. Price and quantity of a commodity
17. $x^2 - 3x^3$
18. 0
19. Both A and R are true and R is the correct explanation of A.
20. Both A and R are true and R is the correct explanation of A

21. $2x - 5 \le x + 2 \le 3x + 8$

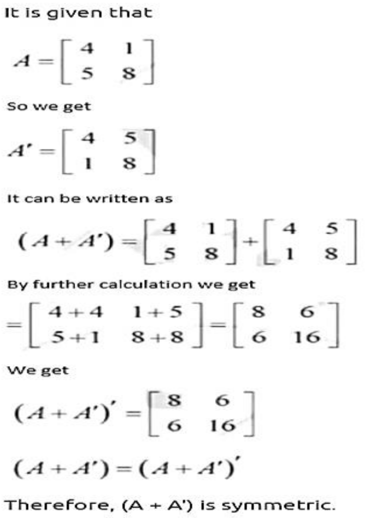
$2x - 5 \le x + 2 \quad , \quad x + 2 \le 3x + 8$

$x \le 7 \qquad , \qquad -2x \le 6 \ , ie\ x \ge -3$

Solution is [-3,7]



22.



It is given that

$A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$

So we get

$A' = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$

It can be written as

$(A + A') = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$

By further calculation we get

$= \begin{bmatrix} 4+4 & 1+5 \\ 5+1 & 8+8 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$

We get

$(A + A')' = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$

$(A + A') = (A + A')'$

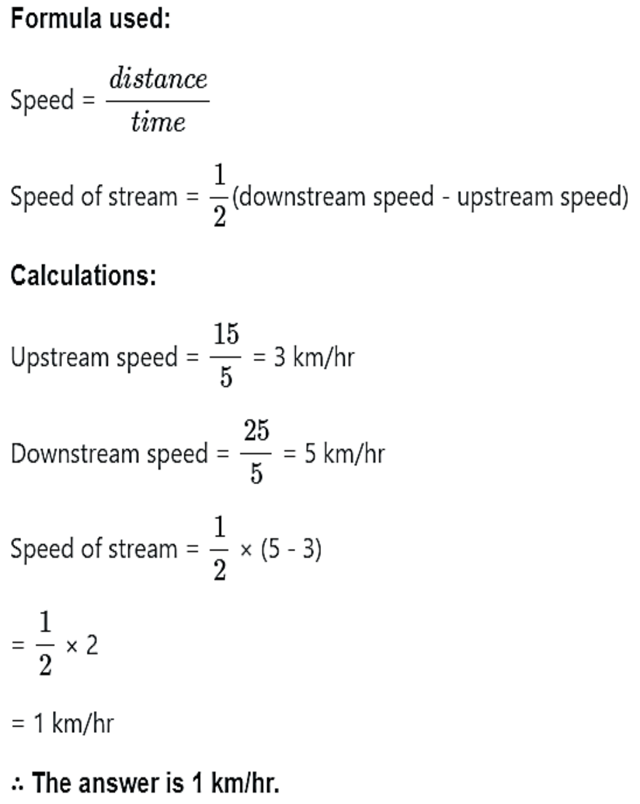
Therefore, (A + A') is symmetric.

OR

$$\begin{bmatrix}1\\6\\6\end{bmatrix}$$

23.MAX Z= x+3y subject to constrains $x + 3y \leq 12$,$3x + y \leq 12$, $x \geq 0, y \geq 0$

24.



Formula used:

Speed = $\frac{distance}{time}$

Speed of stream = $\frac{1}{2}$(downstream speed - upstream speed)

Calculations:

Upstream speed = $\frac{15}{5}$ = 3 km/hr

Downstream speed = $\frac{25}{5}$ = 5 km/hr

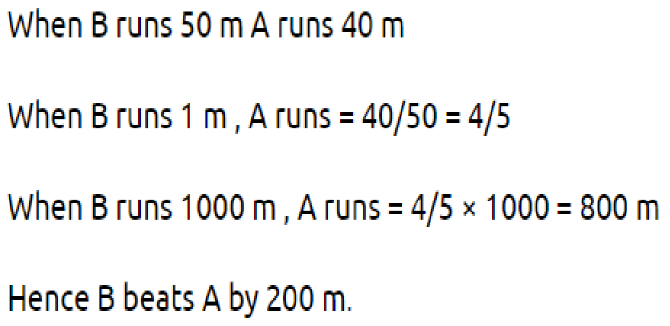
Speed of stream = $\frac{1}{2}$ × (5 - 3)

= $\frac{1}{2}$ × 2

= 1 km/hr

∴ The answer is 1 km/hr.

OR



When B runs 50 m A runs 40 m

When B runs 1 m , A runs = 40/50 = 4/5

When B runs 1000 m , A runs = 4/5 × 1000 = 800 m

Hence B beats A by 200 m.

25.

$x = 4t$

Differentiating w.r.t. t, we get,

$\frac{dx}{dt} = 4$

$y = \frac{4}{t}$

Differentiating w.r.t. t, we get,

$\frac{dy}{dt} = \frac{-4}{t^2}$

Thus, $\frac{dy}{dx} = \frac{-4/t^2}{4} = -\frac{1}{t^2}$

26.

Let $I = \int \frac{2x+1}{(x+1)(x-2)}\,dx$

Let $\frac{2x+1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$

$\therefore 2x + 1 = A(x - 2) + B(x + 1)$ ....(i)

Putting $x = -1$ in (i), we get

$2(-1) + 1 = A(-3) + B(0)$

$\therefore -1 = -3A$

$\therefore A = \frac{1}{3}$

Putting $x = 2$ in (i), we get

$2(2) + 1 = A(0) + B(3)$

$\therefore 5 = 3B$

$\therefore B = \frac{5}{3}$

$\therefore \frac{2x+1}{(x+1)(x-2)} = \frac{\frac{1}{3}}{x+1} + \frac{\frac{5}{3}}{x-2}$

$\therefore I = \int \left(\frac{\left(\frac{1}{3}\right)}{x+1} + \frac{\left(\frac{5}{3}\right)}{x-2} \right) dx$

$\therefore \frac{1}{3} \int \frac{1}{x+1} dx + \frac{5}{3} \int \frac{1}{x-2}\,dx$

$\therefore I = \frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$

OR

$\int(1+x)\log x\,dx =$ $\log x \cdot \int (1+x)\,dx - \int\left(\frac{d}{dx}(1+x)\cdot\int \log x\right) dx$

$$= \log x\left[x + \frac{x^2}{2}\right] - \int\left(x + \frac{x^2}{2}\right)\frac{1}{x}$$

$$= \log x\left[x + \frac{x^2}{2}\right] - x + \frac{x^2}{4} + C$$

27.

a − 8 = 1

⇒ a = 9

3b = −2

⇒ b = − 2/3

−c + 2 = −28

⇒ c = 30

⇒ 2a + 3b − c = − 14

28.

$f'(x) = x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$

⇒ x = 1, 2, 3

Strictly increasing in (1, 2) ∪ (3, ∞)

Strictly decreasing in (−∞, 1) ∪ (2, 3)

OR

Let r be the radius, s be the surface area and V be the volume of the spherical balloon.

Then, $\frac{ds}{dt}$ = 2 cm^2/sec, r = 6 cm ....[Given]

$$\frac{ds}{dt} = 4\pi(2r) \cdot \frac{dr}{dt}$$

$$\therefore 2 = 8\pi r \cdot \frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{1}{4\pi r} \quad ....(i)$$

Now, $V = \frac{4}{3}\pi r^3$

Differentiating w.r.t. t, we get

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3\pi^2\right) \cdot \frac{dr}{dt}$$

$$= 4\pi r^2\left(\frac{dr}{dt}\right)$$

$$= 4\pi r^2 \cdot \frac{1}{4\pi r} \quad ....[\text{From (i)}]$$

$$= r$$

$$\therefore \frac{dV}{dt} = 6 \text{ cm}^3/\text{sec}$$

29. 50 – 8x =5+x

45=9x

x=5

$p_0 = 50 - 40 = 10$

$$\int_0^5 50 - 8x \, dx - 50$$

$CS =$

$$= \left[50x - \frac{\overset{4}{\cancel{8}}x^2}{2}\right]_0^5 - 50$$

$$= 250 - 100 - 50 = 100$$

$$PS = 50 - \int_0^5 5 + x \, dx$$

$$= 50 - \left[5x + \frac{x^2}{2}\right]_0^5 = 50 - \left[25 + \frac{25}{2}\right]$$

$$= 50 - 25 - \frac{25}{2} = 12 \cdot 5$$

30.

Let the time taken by pipe A to fill the tank be x minutes

Time is taken by pipe B to fill the tank = x+5 minutes

So, 1/x + 1/(x+5) = 1/6

⇒ x = 10

Thus, time taken by B alone to fill the tank is 10+5, i.e., 15 minutes

31.

Given,

3/(x-2) < 1

$$\Rightarrow \frac{3}{x-2} - 1 < 1 - 1$$

$$\Rightarrow \frac{3}{x-2} - 1 < 0$$

$$\Rightarrow \frac{3-(x-2)}{x-2} < 0$$

$$\Rightarrow \frac{3-x+2}{x-2} < 0$$

$$\Rightarrow \frac{5-x}{x-2} < 0$$

$$\Rightarrow \frac{x-5}{x-2} > 0$$

For this inequation to be true,

There are two possible cases.

i. x – 5 > 0 and x – 2 > 0

⇒ x – 5 + 5 > 0 + 5 and

x – 2 + 2 > 0 + 2

⇒ x > 5 and x > 2

∴ x ∈ (5, ∞) ∩ (2, ∞)

However,

(5, ∞) ∩ (2, ∞) = (5, ∞)

Hence,

x ∈ (5, ∞)

ii. x – 5 < 0 and x – 2 < 0

⇒ x – 5 + 5 < 0 + 5 and

x – 2 + 2 < 0 + 2

⇒ x < 5 and x < 2

∴ x ∈ (–∞, 5) ∩ (–∞, 2)

However,

(–∞, 5) ∩ (–∞, 2) = (–∞, 2)

Hence,

x ∈ (–∞, 2)

Thus,

The solution of the given inequation is (–∞, 2)∪ (5, ∞).

32. Revenue R = px=$200x + 20x^2 - x^2$

$1) MR = \frac{d}{dx}(200x + 20x^2 - x^2) = 200 + 20\frac{x^3}{3} - \frac{x^3}{3}$

2)6533.33

3)8629.66-6533.33=2096.33

33.

Given system of equations is

$3x + 2y - 2z = 3$

$x + 2y + 3z = 6$

and $2x - y + z = 2$

In the form of AX = B

$$= \begin{bmatrix} 3 & 2 & -2 \\ 1 & 2 & 3 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$$

For A^{-1}, $|A| = |3(5) - 2(1-6) + (-2)(-5)|$

$= |15 + 10 + 10| = |35| \equiv 0$

$\therefore A_{11} = 5, A_{12} = 5, A_{13} = -5$

$A_{21} = 0, A_{22} = 7, A_{23} = 7$

$A_{31} = 10, A_{32} = -11, A_{33} = 4$

$$\therefore \operatorname{adj} A = \begin{vmatrix} 5 & 5 & -5 \\ 0 & 7 & 7 \\ 10 & -11 & 4 \end{vmatrix}^T = \begin{vmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{vmatrix}$$

$$\text{Now } A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{35}\begin{vmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{vmatrix}$$

For $X = A^{-1}B$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{35}\begin{vmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{vmatrix} = \frac{1}{35}\begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore x = 1, y = 1$ and $z = 1$

OR

D=91

$D_1 = 91$

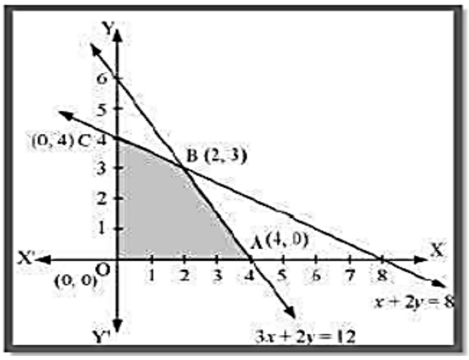
$D_2 = 182$

$D_3 = 91$

x=1 ,   y=2 ,   z=1

34.

The feasible region determined by the system of constraints, x +2y≤ 8, 3x+2y≤12,x ≥ 0, and y ≥ 0, is as follows.



The corner points of the feasible region are O (0, 0), A (4, 0), B (2, 3), and C (0, 4).
The values of Z at these corner points are as follows

Corner point	Z = −3x + 4y	
O(0, 0)	0	
A(4, 0)	−12	→ Minimum
B(2, 3)	6	
C(0, 4)	16	

Therefore, the minimum value of Z is −12 at the point (4, 0).

35.

Solution. $AB = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} [-2 \quad -1 \quad -4]$

$$= \begin{bmatrix} (-1).(-2) & (-1).(-1) & (-1).(-4) \\ 2.(-2) & 2.(-1) & 2.(-4) \\ 3.(-2) & 3.(-1) & 3.(-4) \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{bmatrix}$$

$$\Rightarrow \quad (AB)' = \begin{bmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{bmatrix}' = \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

$$\text{Also } B'A' = [-2 \quad -1 \quad -4]' \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}' = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix} [-1 \quad 2 \quad 3]$$

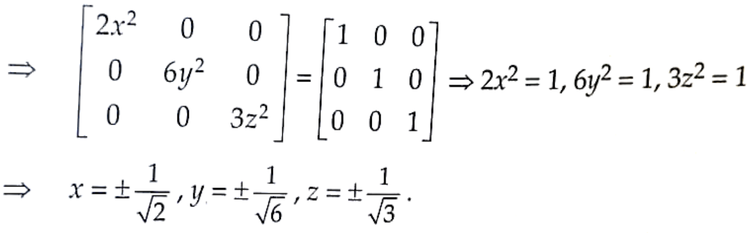
$$= \begin{bmatrix} (-2).(-1) & (-2).2 & (-2).3 \\ (-1).(-1) & (-1).2 & (-1).3 \\ (-4).(-1) & (-4).2 & (-4).3 \end{bmatrix} = \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

From (*i*) and (*ii*), we find that $(AB)' = B'A'$.

OR

Solution. Given $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

$$\text{Now } A'A = I_3 \Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow 2x^2 = 1, 6y^2 = 1, 3z^2 = 1$$

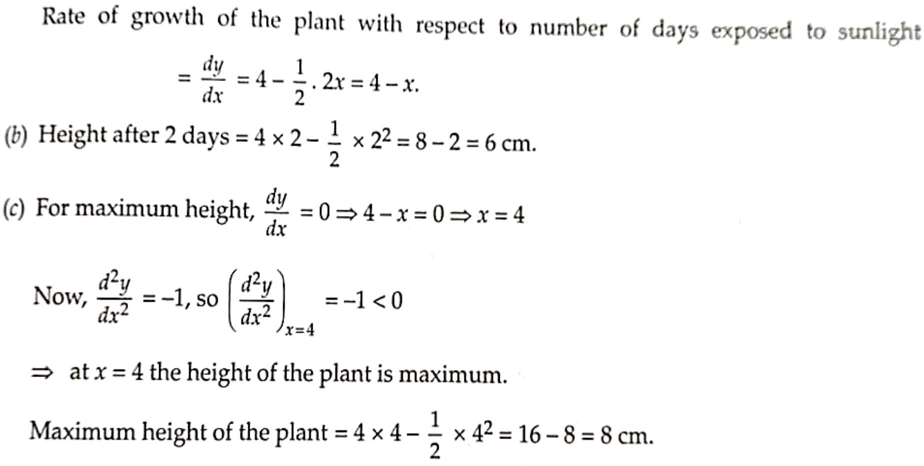
$$\Rightarrow \quad x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}.$$

36.(a)3

(b) 1

(c)2

37.



Rate of growth of the plant with respect to number of days exposed to sunlight

$$= \frac{dy}{dx} = 4 - \frac{1}{2}.2x = 4 - x.$$

(*b*) Height after 2 days $= 4 \times 2 - \frac{1}{2} \times 2^2 = 8 - 2 = 6$ cm.

(*c*) For maximum height, $\frac{dy}{dx} = 0 \Rightarrow 4 - x = 0 \Rightarrow x = 4$

Now, $\frac{d^2y}{dx^2} = -1$, so $\left(\frac{d^2y}{dx^2}\right)_{x=4} = -1 < 0$

⇒ at $x = 4$ the height of the plant is maximum.

Maximum height of the plant $= 4 \times 4 - \frac{1}{2} \times 4^2 = 16 - 8 = 8$ cm.

38. (a)(0,8)

(b) -32

(c)point-(5,0) value -15